Evolutionary dynamics of time-resolved social interactions

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Outline

- Short Introduction on Evolutionary Game Theory
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- Time Varying Graphs
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- Datasets
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- Results
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- Datasets
- Results
- Conclusions.
Motivation

Foreword

Dynamical processes acting on time varying graphs behave differently than on static graphs.

Motivation

Question:

Does time resolution affects the classical results about the enhancement of cooperation driven by static networks?
The game: Social Dilemma

Consider a pairwise interaction where individuals face a social dilemma between two possible strategies: *Cooperation* (C) and *Defection* (D). Such dilemmas can be encoded into a two-parameter game described by the payoff matrix:

\[
\begin{pmatrix}
C & D \\
C^T & S \\
D & T & P
\end{pmatrix} = \begin{pmatrix}
C & D \\
1 & S \\
T & 0
\end{pmatrix},
\]
Short Introduction on Evolutionary Game Theory

Mean field case

\[ \begin{pmatrix} C & D \\ C & 1 & S \\ D & T & 0 \end{pmatrix} \]

We consider three different kinds of social dilemmas, namely:

- Harmony Game (HG)
- Chicken Game (CG)
- Stag Hunt (SH)
- Prisoner's Dilemma (PD)
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Strategy Update

After all the individuals have played with all their neighbors in the network, they update their strategies as a result of an evolutionary process. To update the strategies of agents we consider the so-called Fermi Rule:

\[
P_{i \rightarrow j} = \frac{1}{1 + e^{-\beta(p_j - p_i)}} ,
\]  

(1)
Time Varying Graphs

\[ G1 \quad G2 \quad G3 \quad G4 \]

\[ \tau \]

\[ n \tau = \Delta t \]
Datasets

MIT Reality Mining

Data of proximity interactions collected through the use of Bluetooth-enabled phones distributed to a group of 100 users, composed by 75 MIT Media Laboratory students and 25 faculty members recorded over a period of about six months.

\[
\begin{array}{|c|c|c|c|c|}
\hline
M & N & \tau & E^* & \langle k \rangle_{agg} \\
\hline
41291 & 100 & 5 \text{ min} & 2114 & 42 \\
\hline
\end{array}
\]

The data set consists of proximity measurements collected during the IEEE INFOCOM’06 conference held in a hotel in Barcelona between 23-rd and 29-th of April 2006.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$N$</th>
<th>$\tau$</th>
<th>$E^*$</th>
<th>$\langle k \rangle_{agg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2880</td>
<td>78</td>
<td>2 min</td>
<td>2730</td>
<td>70</td>
</tr>
</tbody>
</table>

Datasets

Experimental setup:

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- Initial fraction of cooperators $f_c(0) = 0.5$ randomly distributed.
- Payoff parameters $T \in [1, 2]$  $S \in [-1, 1]$.
- Two kind of time sequence: the real one and a randomized version.
- Averaged over 50 different realizations.
Time series

Reality

Infocom
We measure the cooperation level as:

\[ \langle C(T, S)_{\Delta t} \rangle = \frac{1}{Q} \sum_{i=1}^{Q} \frac{N_i^c}{N} , \]
Overall level of cooperation \( C_{tot}(\Delta t) \)

\[
C_{tot}(\Delta t) = \frac{1}{C_{tot}(M_T)} \int_{-1}^{1} \int_{0}^{2} C(T, S) dS dT
\]
Summing up . . .

Take home messages

- The level of cooperation achievable on time-varying graphs crucially depends on the temporal resolution, i.e. on the length of the aggregation interval used to construct each graph.
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- Trying to find bigger datasets.
- Use different randomization methodologies?
Time series

- $\sigma_{on}$ → Contact duration.
- $\sigma_{off}$ → Inter-contact time.