Co-evolution of strategies and update rules in the prisoner’s dilemma game on complex networks

Alessio Cardillo

Department of Physics of Condensed Matter – University of Zaragoza
and
Institute for Biocomputation and Physics of Complex Systems (BIFI), Zaragoza, Spain
http://bifi.es/~cardillo/

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Summary

- Introduction
- Game theory
- Update Rules
- Complex Networks
- Results
- Conclusions
Introducing myself . . .

Actually I am working on . . .

- Dynamical processes on networks;
- Evolutionary game theory on networks;
- Emergence of collective behaviours (i.e. cooperation);

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Co-evolution of strategies and update rules in the prisoner’s dilemma game on complex networks

Alessio Cardillo\textsuperscript{1}, Jesús Gómez-Gardeñes\textsuperscript{2,3,6}, Daniele Vilone\textsuperscript{4} and Angel Sánchez\textsuperscript{3,4,5}

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Angel Sánchez: Dept. of Mathematics, University Carlos III, Madrid, Spain
Jesús Gómez-Gardeñes: Dept. of Physics of Condensed Matter and BIFI, Zaragoza, Spain
Daniele Vilone: Dept. of Mathematics, University Carlos III, Madrid, Spain
Introduction to Prisoner’s Dilemma (PD) game

Situation: Two bank robbers have been arrested ...
From a mathematical point of view we can *describe* the game through the *payoff matrix* as:

\[
\begin{pmatrix}
C & D \\
\mathcal{R} & \mathcal{S} \\
\mathcal{T} & \mathcal{P}
\end{pmatrix}
\]
such that:  
\( \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S} \).
Introduction to Prisoner’s Dilemma (PD) game

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\[
\begin{pmatrix}
C & D \\
C & R & S \\
D & T & P
\end{pmatrix}
\]

such that: \(T > R > P > S\).

**Nash equilibrium**

The Nash equilibrium of PD game is the *defection* state. The problem with Nash equilibrium is that:

- Players are not “smart” (they are not able to calculate the Nash equilibrium);
- Players do not have “full knowledge” (they do not know the structure of payoff matrix);
Introduction to Prisoner’s Dilemma (PD) game

Game theory is not enough

- In general, people do not play only once but many times;
- They learn after each round and try to choose a strategy which ensure them the best success in the next one (payoff driven selection);
- Humans are not always “fully rational” because sometimes they make counterintuitive choices;
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Evolutionary game theory

These objections could find a solution under the evolutionary game theory postulated by Maynard Smith and Price in 1973.
Game alone is not enough

Once player’s strategies are defined, one has to define also how players update their strategies during the game, i.e. their Update Rules.
Update Rules

Definitions

Replicator Dynamics (REP): Each agent $i$ chooses one of his neighbors at random, say $j$, and compares their payoffs. If $f_j > f_i$ agent $i$ will copy strategy and update rule of $j$ with probability:

$$\Pi \propto f_j - f_i.$$
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Unconditional Imitator (UI): Each agent $i$ looks at all his neighbors $j$, choose the one with the highest payoff and if $f_j > f_i$ he will copy both strategy and update rule of $j$ or maintains its own otherwise.
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**Unconditional Imitator (UI):** Each agent $i$ looks at all his neighbors $j$, chooses the one with the highest payoff and if $f_j > f_i$ he will copy both strategy and update rule of $j$ or maintains its own otherwise.

**Moran Rule (MOR):** Each agent $i$ chooses one of his neighbors proportionally to his payoff and changes his state to the one of the chosen one.
Update Rules

In particular:

- REP → is stochastic with partial information;
- UI → is deterministic with full information;
- MOR → is stochastic with full information;
Update Rules

An example:

(a) REP

(b) MOR

(c) UI
Interaction patterns

Once the game is fully set-up, we have to decide how the players interact between them. Mean-field and regular lattices are two paradigmatic examples but complex topologies are best suited to represent a real-scenario.
Two of the most common topologies used are: **Scale Free (SF)** and **Erdős-Rényi (ER)** graphs.
Complex Networks

Erdős-Rényi & Lattice

G. Abramson and M. Kuperman, PRE, 63, 030901(R) (2001)

Scale-free

F.C. Santos and J.M. Pacheco, PRL, 95, 098104 (2005)
Scale-free networks are very important because many real systems display such kind of structure.
Interaction patterns

In order to consider topologies spanning from ER networks to SF ones, we used the Interpolation Model of Gardeñes et al.

Papers in literature show that the outcome of evolutionary game on complex networks, in terms of cooperative behavior, strongly depends on topology but also on the update rule used.
The question is: Why Co-evolution?

- Papers in literature show that the outcome of evolutionary game on complex networks, in terms of cooperative behavior, strongly depends on topology but also on the update rule used.
- Since there are not any a-priori reasons to fix the update rule or to impose one over the others, we treat update rule in the same manner as strategy: we let it evolve, allowing the system itself choose what he “likes” most.
The question is: Why Co-evolution?

- Papers in literature show that the outcome of evolutionary game on complex networks, in terms of cooperative behavior, strongly depends on topology but also on the update rule used.
- Since there are not any a-priori reasons to fix the update rule or to impose one over the others, we treat update rule in the same manner as strategy: we let it evolve, allowing the system itself choose what he “likes” most.
- We want to see if the coexistence of different update rules, in association with different underlying topologies, changes the overall cooperative behavior present in literature.
Experimental setup

**Game:** Weak Prisoner’s Dilemma with payoff-matrix given by:

\[
\begin{pmatrix}
C & D \\
C & (1, 0) \\
D & (b, 0)
\end{pmatrix}
\]

with \( b \in [1, 2] \).

**Update Rule:** REP, UI, MOR;

**Topologies:** SF (\( \alpha = 0 \)), Intermediate (\( \alpha = 0.5 \)), ER (\( \alpha = 1 \))

\( N = 5000 \) and \( \langle k \rangle = 6 \);

**Other information:**

- Pairwise game with two different update rules per “realization”;
- Initial fraction of players with a certain rule \( x_{\text{rule}}(0) \in [0, 1] \);
- Initial fraction of cooperators and defectors \( f_C(0) = f_D(0) = 0.5 \);
- Payoff does not accumulate through gaming and the update of the strategies is synchronous.
- Dynamic evolution of the system = 4000 game rounds;
- All simulations averaged over 100 different realizations for each set of parameters;
Results are analyzed looking at the behavior of two quantities (with respect to the temptation parameter $b$):

- **Average cooperation level in the asymptotic regime** $\langle C \rangle = \frac{nc}{N}$;
Results are analyzed looking at the behavior of two quantities (with respect to the temptation parameter $b$):

- Average cooperation level in the asymptotic regime $\langle C \rangle = \frac{nc}{N}$;
- Average final fraction of players with a certain rule $\langle x_{\text{rule}} \rangle = \frac{n_{\text{rule}}}{N}$. 
Evolutionary dynamics on networks is very different from the mean-field problem;

Evolution on Scale-Free networks allows survival of cooperation even when the temptation to defect is relatively high;

Co-evolution shows that it is possible to obtain relatively large cooperation values when two update rules coexist in contrast with the single rule case. In particular we observe this in:

- REP vs UI in ER networks;
- MOR vs UI in SF networks;
Conclusions

- Evolutionary dynamics on networks is very different from the mean-field problem;
- Evolution on Scale-Free networks allows survival of cooperation even when the temptation to defect is relatively high;
- Co-evolution shows that it is possible to obtain relatively large cooperation values when two update rules coexist in contrast with the single rule case. In particular we observe this in:
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  + MOR vs UI in SF networks;

Further Developments

- Consider different time scales for the game and change of the update rule processes;
- Study the scenario in which three rules are used simultaneously;
- Consider the use of other update rules (e.g. Fermi rule) and/or other games (e.g. Stag Hunt);
Start with a seed network with $m_0$ nodes fully connected between them.
Details on the interpolation model

1. Start with a *seed network* with $m_0$ nodes fully connected between them;
2. Add $U = N - m_0$ nodes, each with $m \leq m_0$ links;
Details on the interpolation model

**Interpolation Model**

1. Start with a *seed network* with $m_0$ nodes fully connected between them;
2. Add $U = N - m_0$ nodes, each with $m \leq m_0$ links;
3. Each link has a probability $\alpha$ to be attached *randomly* to one of the $N - 1$ other nodes, and a probability $1 - \alpha$ to be attached using the preferential attachment model of Barabasi and Albert;
Details on the interpolation model

A. Cardillo (Univ. of Zaragoza)

Co-evol. of strategy & update rule in PD games

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