Velocity-enhanced Cooperation of Moving Agents playing Public Goods Games

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In this Brief Report we study the evolutionary dynamics of the Public Goods Game in a population of mobile agents embedded in a 2-dimensional space. In this framework, the backbone of interactions between agents changes in time, allowing us to study the impact that mobility has on the emergence of cooperation in structured populations. We compare our results with a static case in which agents interact on top of a Random Geometric Graph. Our results point out that a low degree of mobility enhances the onset of cooperation in the system while a moderate velocity favors the fixation of the full-cooperative state.

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Despite its ubiquity in nature and human societies, the survival of cooperative behaviors among unrelated agents (from bacteria to humans) when defection is the most advantageous strategy constitutes one of the most fascinating theoretical challenges to the predictions of Evolutionary Theory [1,2]. Recently, it has been pointed out that the integration of the microscopic patterns of interactions among the agents composing a large population into the evolutionary setting provides a way out for cooperation to survive in paradigmatic scenarios such as the Prisoner’s Dilemma (PD) game [3]. Since this latter seminal work, a large body of studies have addressed the problem of linking the emergence of cooperative behavior with the structural patterns of the backbone of interactions, typically encoded as a complex network. This wealth of works has brought a new discipline, known as Evolutionary Graph Theory, joining tools and methods from statistical mechanics of complex networks and evolutionary game dynamics [4,5].

The structural features studied span many of the real patterns displayed by social networks [6,8], such as the small-world effect [9], their scale-free pattern for the number of contacts per individual [10,11], the presence of clustering [12,13] or their modular architecture [14].

Although the above studies mostly focus on the PD game, other paradigmatic settings have also been studied on top of network substrates, such as the Public Goods Game. The Public Goods Game (PGG) is seen as the natural extension of a PD game when passing from pairwise to $n$-person games. Thus, the recent attention has focused on unveiling to which extent the results found in the context of the PD game apply when moving to a more refined scenario. In fact, the seminal work of Santos et al. [17] showed that the scale-free architecture of the underlying network of contacts again favors the resilience of cooperation in contrast to what is found in well-mixed (mean-field) populations. Many other works have continued this line of research by exploring the networked version of the PGG [18-24]. Moreover, as the PGG formulation introduces two structural scales, namely individuals (considered as the nodes of a network) and the groups within which they interact (treated as a set of subgraphs embedded in the original network), it has been shown that the structure of the mesoscale defined by the groups also play an important role for the success of cooperation [25-28].

The assumption of a static graph that maps social ties, although still a coarse grained picture of the microscopic interactions, provides with a useful approach for studying the dynamics of large social systems. However, when moving to smaller scales one has to consider additional microscopic ingredients that may influence the collective outcome of social dynamics. One of these ingredients is the mobility of individuals, a topic that has recently attracted a lot of attention, and that has been tackled from different perspectives. The range of studies in which mobile agents have been addressed spans from pure empirical studies [29-31], to theoretical ones that focus on the role that mobility patterns have on different dynamical processes. This latter approach covers different dynamical settings such as disease spreading [32], synchronization [33,34] and evolutionary dynamics [35-37] in the context of the PD game.

In this Brief Report we follow the setting introduced in [35] in which a population of $N$ agents moves on a 2-dimensional space. Simultaneously to the movement of the agents we consider that a PGG is played. To this end, the movement dynamics is frozen at equally spaced time steps and each node engages its closest neighbors to participate in a group in which a PGG is played. Obviously, the mobility of individuals turns the usual static backbone of interactions into an time-evolving one, opening the door to novel effects on the evolution of cooperation. We will start our discussion by making a comparison between a static configuration in which agents do not move (planar graph) with an equivalent network model in which spatial effects are absent. Second, we will consider the effect of motion on the promotion of cooperation by studying its evolution as a function of the velocity of agents. Our results point out that a low degree of mobility enhances the onset of cooperation in the PGG while a moderate velocity favors the fixation of cooperation in the system.

We start by introducing the dynamical setting in which the evolutionary dynamics of the PGG is implemented. Our population is composed of a set of $N$ agents living in the area inside a square with side length $L$. Thus, the density of individuals is defined as $\rho = N/L^2$. Both the density and the number of agents remain constant along our simulations. Our agents are initially scattered at random on top of the surface by using two independent random variables uniformly distributed in the in-
Once the RGG is formed, each of the agents defines a group together with her nearest neighbors in the RGG in which one PGG is played. In this way, we follow the method introduced in [17], so that an agent with \( k_i(t) \) neighbors in the instant RGG plays the PGG within \( k_i(t) + 1 \) groups. In each of the groups she participates in, a cooperator player contributes an amount \( c \) while a defector does not contribute. As a result the total payoff accumulated by an agent \( i \) at time \( t \) reads:

\[
P_i(t) = \sum_{j=1}^{N} \left( A_{ij}^t + \delta_{ij} \right) \sum_{k=1}^{N} \left( A_{ij}^t + \delta_{ij} \right) s_k(t) \theta_{ij} - \sum_{k=1}^{N} \left( k_i(t) + 1 \right) s_i(t) c.
\]

(3)

After each round of the PGG is played each of the agents can update her strategy. To this aim, an agent \( i \) chooses one of her instant neighbors \( j \) at random and with some probability \( \Pi[s_i(t+1) = s_j(t)] \) will take the strategy of \( j \) during the next round of the PGG. The former probability reads as follows:

\[
\Pi[s_i(t+1) = s_j(t)] = \frac{\Theta[P_i(t) - P_j(t)]}{M[k_i(t), k_j(t)]},
\]

(4)

where \( \Theta(x) = x \) when \( x > 0 \) while \( \Theta(x) = 0 \) otherwise, and \( M(k_i, k_j) \) is the maximum possible payoff difference between two players with instant degrees \( k_i(t) \) and \( k_j(t) \). In our simulations, we let co-evolve both movement and evolutionary dynamics during \( 5 \cdot 10^3 \) time steps. We take the first \( 25 \cdot 10^3 \) steps as a transient period while the degree of cooperation of the system is measured during the second half of the simulations as:

\[
\langle c \rangle = \frac{1}{T} \sum_{t = \tau}^{\tau + T} \sum_{i=1}^{N} s_i(t),
\]

(5)

with both \( \tau = T = 25 \cdot 10^3 \). The results reported below are averaged over different realizations (typically 50).

We start our analysis by considering the static case in which the velocity of the agents is set to \( v = 0 \). In this case, the interaction RGG is fixed from the initial configuration while only the strategies of agents evolve. A RGG is described by a Poissonian distribution, \( P(k) = \langle k \rangle^k e^{-\langle k \rangle} / k! \), for the probability of finding a node connected to \( k \) neighbors. This distribution corresponds to a rather homogeneous architecture in which the dispersion around the mean degree, \( \langle k \rangle \), is rather small. The same pattern for the degree distribution \( P(k) \) is obtained for the typical Erdős-Rényi random network model. However, the main differences between RGG and ER networks relies on the clustering coefficient, \( i.e. \) the probability that two nodes with a common neighbor share a connection. While in the case of ER graphs clustering vanishes as \( N \to \infty \), the geometric nature of RGG boosts the density of triads leading to a finite and large clustering coefficient. This difference has been shown to be of relevance for the synchronization properties of RGG as compared to ER graphs [38]. Thus, in order to unveil the role that the topological patterns of a RGG plays in the outcome of the evolutionary dynamics of the PGG we compare its results with those obtained in ER graphs. In order
to make such comparison meaningful, both graphs have the same number of elements \( N \) and the same average degrees \( \langle k \rangle \) which, in the case of a RGG is determined by the radius \( R \) and the density of agents considered: \( \langle k \rangle = \rho \pi R^2 \).

The results of the above analysis are shown in Fig. 2, where we represent the dependency of the average level of cooperation in the system \( \langle c \rangle \) with respect to the enhancement factor, \( r \), for both RGG and ER graphs. As expected, for low values of \( r \) defection dominates the system while for large \( r \) cooperation prevails. Between these two asymptotic regimes the transition from defection to cooperation occurs \( (5 \leq r \leq 8) \) pointing out slight differences between RGG and ER graphs. In this region we observe that ER networks promote cooperation slightly more than RGG for which the transition curve towards full cooperation goes slower. As anticipated above, the reason why the transition in the RGG is smoother than in ER relies on the different clusterization of nodes in the two systems. Nevertheless, the onsets of both transitions are roughly the same.

We now focus on the impact that the motion of agents has on the level of cooperation with respect to the static case. Thus, from now on, we consider that agents move with constant velocity \( v \) following the rules given by Eqs. 1 and 2. Moreover, we set the value of the enhancement factor \( r \) to be in the region for which the transition from full defection to cooperation occurs in the static case, namely \( r = 5.75 \). Then, we monitor the degree of cooperation \( \langle c \rangle \) sustained in the system as a function of the velocity by increasing it from small to large values. The result of this analysis is shown in Fig. 3 together with the value (dashed line) for \( \langle c \rangle \) in the static limit corresponding to \( r = 5.75 \). From this figure we observe a rise-and-fall of cooperation so that when the velocity \( v \) increases from very small values, the average level of cooperation in the stationary state increases significantly, reaching its maximum value for \( v \approx 2 \cdot 10^{-2} \). From this point on, the increase of \( v \) leads to the decay of cooperation so that \( \langle c \rangle = 0 \) beyond \( v \approx 10^{-1} \). The fall of cooperation for large values of the velocity of agents is a quite expected result: as the velocity increases one approaches the well-mixed scenario for which cooperation is suppressed provided \( r \) is less than the typical size of groups in which the PGG is played. In our case \( \langle k \rangle = 6 \) so that groups are typically composed of 7 agents, being this value larger than the enhancement factor used in Fig. 3 \( (r = 5.75) \). However, the rise of cooperation for small values of \( v \) is somehow striking. Such a behavior indicates that there exists an optimal range for the velocity of agents in which cooperation is promoted with respect to the static case.

A more extensive analysis on the effects of motion is found in Fig. 4, where a detailed exploration of the \( (v, r) \)-parameter space is shown together with the cooperation level in the static case (bottom part of the panel) as obtained from the corresponding curve in Fig. 2. This panel confirms the results obtained in Fig. 3 and provides a more complete picture about the enhancement of cooperation produced by the mobility of agents. First, by comparing the bottom \( (v = 0) \) and upper \( (v = 10^{-1}) \) parts of the panel we observe that a large value of the velocity decreases the cooperation level of the static system. In particular, let us note that the transition region in the limit of large velocity is placed around \( r \approx 7 \) thus recovering the well-mixed prediction. However, the relevant results are found between the static and large velocity limits. The effects of mobility in this region affect both the onset of the transition towards cooperation, i.e. the minimum value of \( r \) for which a nonzero level of cooperation is observed, and its fixation, i.e. the minimum value of \( r \) for which the system reaches the absorbing state corresponding to full-cooperation. First, we observe that even for very low values of \( v \) the onset of cooperation is anticipated with respect to the static case at the expense of having a broader transition towards full-cooperation as compared to the static RGG. However, when the velocity level is further increased, the transition becomes sharper and both the onset and the fixation of the full-cooperative state occur before with respect to the static case.
Summing up, the results presented in this Brief Report point out that the mobility of the agents playing a PGG enhances cooperation provided their velocity is moderate. This enhancement is obtained by comparing the outcome of the evolutionary dynamics of the PGG with the results obtained in the static case, here described as a structured population of agents in which the backbone of interactions is defined by a RGG. The addition of the random movement of agents produces the evolution in time of the original RGG, being the rate of creation and deletion of links controlled by the velocity of agents. As the velocity increases we observe an optimal operation regime in which both the onset of cooperation and the fixation of cooperation in the system are enhanced.

Acknowledgments

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FIG. 4: (color online) Cooperation level $\langle c \rangle$ as a function of the velocity of the agents, $v$, and the enhancement factor of the PGG, $r$. The system has a density of players $\rho = 2.0$ and the instant RGG is constructed with $R = 1.0$. The values of the velocities span in the interval $[10^{-5}; 10^{-1}]$. Results are averaged over 50 different realizations for each couple $(r, v)$ of parameters explored. The static case is represented in the bottom part of the panel (below the white line).

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